

Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Mathematics - II

Subject Code: 4SC02MTC1

Branch: B.Sc. (All)

Semester: 2

Date: 09/05/2017

Time: 02:00 To 05:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Find out order and degree of the differential equation (02)

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{d^3y}{dx^3}\right)^2$$

b) Solve: $(D^2 - 1)y = 0$ (02)

c) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$ (02)

d) Find polar form of $1 + \sqrt{3}i$. (01)

e) Find real and imaginary part of $2e^{-\frac{i\pi}{2}}$ (01)

f) $\lim_{n \rightarrow \infty} \sqrt[n]{a} = \underline{\hspace{2cm}}$ where $a > 0$. (01)

g) True/ False. If P.I. = 0 then the general solution of ordinary differential equation is same as the complementary function. (01)

h) Define: Cauchy's sequence. (01)

i) True/ False. Every convergent sequence is bounded. (01)

j) Write equation of ellipsoid. (01)

k) True/ False. If the imaginary part of any complex number is zero then the complex number becomes purely real number. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) State and prove De-moivre's theorem (07)

b) Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$ (05)

c) Simplify: $\frac{(\cos 4\theta - i \sin 4\theta)^4 (\cos 2\theta + i \sin 2\theta)^{-6}}{(\cos 2\theta - i \sin 2\theta)^3 (\cos \theta - i \sin \theta)^{-7}}$. (02)



- Q-3 Attempt all questions (14)**
- a) Prove that $\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$. (05)
- b) Find the roots of the equation $z^4 - 1 = 0$. (05)
- c) Solve: $(D^2 + 2D - 3)y = e^x$. (04)
- Q-4 Attempt all questions (14)**
- a) Using De-moivre's theorem solve $x^7 + x^4 + i(x^3 + 1) = 0$ (07)
- b) Solve: $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 \sin(\log x)$. (07)
- Q-5 Attempt all questions (14)**
- a) Prove that $\cosh^{-1}(z) = \log(z + \sqrt{z^2 - 1})$. (06)
- b) Find real and imaginary part of i^i . (05)
- c) Show that $\sinh(ix) = i \sin x$. (03)
- Q-6 Attempt all questions (14)**
- a) Prove that $I_n = \int \cos^n x dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}$. (06)
- b) Evaluate: $\int_0^1 x^6 \sqrt{1-x^2} dx$. (04)
- c) Solve: $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 4y = \cos x$. (04)
- Q-7 Attempt all questions (14)**
- a) If $f(D)y = e^{ax}$ is given linear differential equation with constant coefficient then prove that $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$, if $f(a) \neq 0$. (06)
- b) Show that the equation $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6})$. (06)
- c) Check whether the sequence $\left\{ \frac{1}{n} \right\}$ convergent or not. (02)
- Q-8 Attempt all questions (14)**
- a) Find equation of lines in which the plane $x + 3y - 2z = 0$ cuts the cone $x^2 + 9y^2 - 4z^2 = 0$. (06)
- b) Find equation of cylinder whose generator are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and guiding curve $x^2 + y^2 = 16, z = 0$. (06)
- c) Evaluate: $\frac{1}{D^2}(x^4)$. (02)

